

Minnesota State High School Mathematics League

Individual Event

2005-06 Event 3A

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

- _____ 1. Find (x,y) , given

$$y = (x - 8)$$
$$(x - 8) + (y + 1) = 5$$

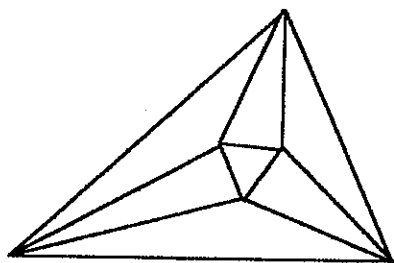
- _____ 2. Find (x,y) , given

$$\frac{3}{8}x - \frac{4}{7}y = 1$$
$$-\frac{2}{3}x + \frac{3}{2}y = 5$$

- _____ 3. The inequalities $0 < x < \frac{3}{2}y$ and $y < k$ describe a region of the xy -plane that has an area of 20. Find k , accurate to three places to the right of the decimal.

- _____ 4. The hour hand and the minute hand coincide sometime between 3 o'clock and 4 o'clock. Express the exact time as a rational number of minutes after 3 o'clock.

Name _____ Team _____



Minnesota State High School Mathematics League

Individual Event

2005-06 Event 3B

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

- _____ 1. Figure 1 shows an isosceles $\triangle ABC$ with vertex $\angle C = 72^\circ$: $\angle EAB = \frac{1}{3}\angle CAB$ and $\angle DBA = \frac{1}{2}\angle CBA$. Find $\angle AFD$.
- _____ 2. Figure 2 shows the exterior angles x_1, x_2, x_3, x_4 , and x_5 of a pentagon with sides of length 1, 2, 3, 4, and 5. What is the sum of $x_1 + x_2 + x_3 + x_4 + x_5$?
- _____ 3. A regular hexagon $ABCDEF$ (Figure 3) has sides of length 2. Through its center K , a line segment JL of length 1 is drawn parallel to AB so that $JK = KL$. How long is DL ?
- _____ 4. The bases of an isosceles trapezoid are 18 and 30. Its height is 8. Using each leg as a diameter, semi-circles are drawn exterior to the trapezoid, and the midpoints of the arcs of the semicircles are designated P and Q . Find the length of PQ .

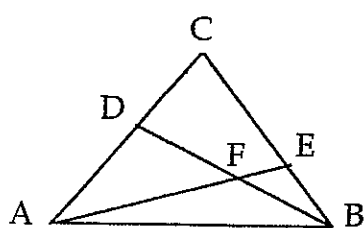


Figure 1

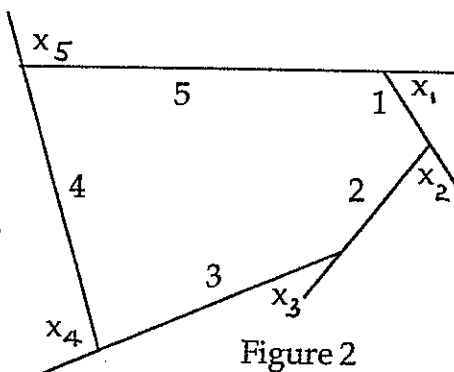


Figure 2

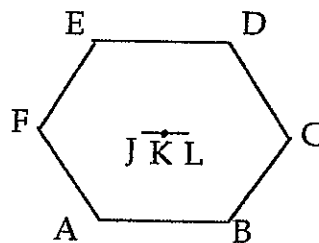
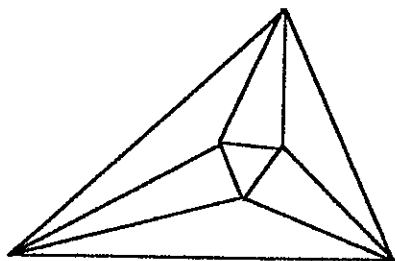


Figure 3

Name _____ Team _____



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Individual Event

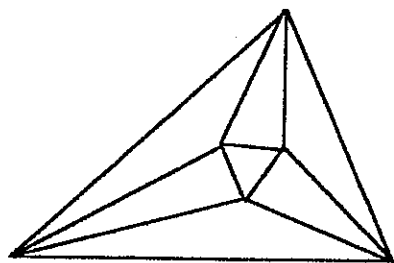
2005-06 Event 3C

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

No calculators are allowed in this event

- _____ 1. The obtuse angle in a triangle has a sine of $\frac{3}{5}$. What is the tangent of this angle?
- _____ 2. In $\triangle ABC$, $\angle A = 30^\circ$, $BC = 2$, $AC = 3$. Find $\sin(\angle B)$.
- _____ 3. Express $(\sqrt{3} + i)^7$ in the form $a\sqrt{3} + bi$ where a and b are rational.
- _____ 4. In a $\triangle ABC$, $AB = 4$, $BC = 5$, and $CA = 6$. Find $\cos(\angle B) - \cos(2\angle C)$.

Name _____ Team _____



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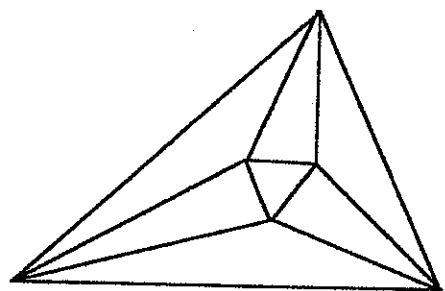
Individual Event

2005-06 Event 3D

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. In each case you are given a graph and a list of equations, one of which corresponds to the graph. Circle what you believe to be the corresponding equation. You have 12 minutes for this event.

1. If $\log_b N = r$, what is $\log_{b^2} N$?
2. Rationalize and simplify $\frac{\sqrt{32} - \sqrt{12}}{\sqrt{6} + 1}$.
3. Express $\log_8 40$ in the form $\frac{a + \log 4}{b \log 2}$, understanding $\log 4$ and $\log 2$ are written to base 10.
4. Given that $0 < M < 1 < N$ and that $4 \log_M N = \log_N M$, what is $\log_N MN$?

Name _____ Team _____



Minnesota State High School Mathematics League

Team Event

2005-06 Meet 3

Each question is worth 4 points. Team members may cooperate in any way, but at the end of twenty minutes, one set of answers is to be submitted. Put answers on the lines provided.

1. Express in the form $(az + b, cz + d, z)$ all solutions to the system $\begin{cases} 2x - 3y - 3z = 4 \\ 3x - 5y + 2z = -3 \end{cases}$

2. Regarding the earth as a sphere, circles of latitude are intersections of the sphere with planes parallel to the equator, and are measured in degrees by the angle x formed by a line drawn from the center of the earth to the circle of latitude (Figure 2). At what latitude is the distance around the earth half of what it is at the equator?

3. From a point outside of a square, the distances to the three nearest vertices are 3, 6, and 7. How long is the side of the square?

4. A conductor notices that his train left the station precisely on the minute; and he notices that the minute hand and the hour hand line up precisely as he passes the 8 mile post at the side of the tracks. If the train averaged 33 miles per hour over the first 8 miles, what time, hours and minutes, did the train leave the station?

5. Find (x, y, z) , given $\begin{cases} \frac{2}{x} - 3y + \frac{4}{z} = 1 \\ -3yz + 4xy^2z - 5xy = -2xyz \\ 5\frac{1}{x} - 2y - 3\frac{1}{z} = 6 \end{cases}$

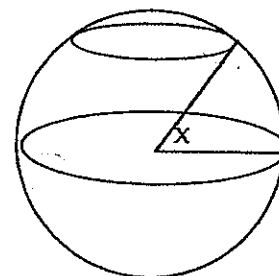


Figure 2

6. Moving counterclockwise around a circle of radius r , the vertices of an inscribed hexagon are $A_1, A_2, A_3, A_4, A_5, A_6$. If $A_1A_2 = A_3A_4 = A_5A_6 = a$ and $A_2A_3 = A_4A_5 = A_6A_1 = b$, express r in terms of a and b .

Team _____